PHYS302 Fall 2023

Homework 7

1. (2 pts) Imagine that we strike two tuning forks, one with a frequency of 440 Hz, the other 443 Hz. What will we hear?

2. (4 pts total)
   a. Show that a standing wave created by two unequal-amplitude waves

$$E_I = E_0 \sin(kx \mp \omega t)$$

and

$$E_R = \rho E_0 \sin(kx \pm \omega t)$$

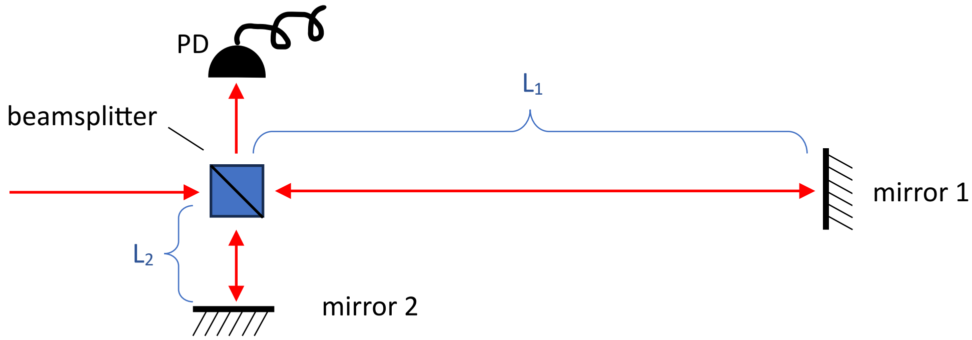
has the form

$$E = 2\rho E_0 \sin kx \cos \omega t + (1-\rho)\, E_0 \sin(kx \mp \omega t).$$

Here ρ is the ratio of the amplitude reflected to the amplitude incident.

   b. Discuss the meaning of the two terms. What happens when ρ = 1?

3. (7 pts) A Michelson interferometer has two arms that split and recombine light on a single beamsplitter:



a. Show that the combined field entering the photodetector (PD) is

$$E = E_{arm1} + E_{arm2} = E_0 e^{i\omega t}\left[e^{i\frac{4\pi L_1}{\lambda}} + e^{i\frac{4\pi L_1}{\lambda}}\right]$$

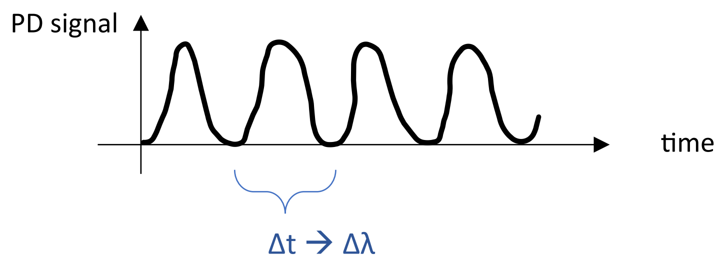
b. Show that the intensity, I, is proportional to

$$I = |E|^2 \propto 1 + \cos\left(\frac{4\pi * (L_1 - L_2)}{\lambda}\right)$$

(Hint 1: $|E|^2$ is the product of E and E*, the complex conjugate of E.)

(Hint 2: cos (z) = [exp(i*z) + exp(-i*z)]/2.)

c. Initially, we have laser light in the interferometer of wavelength $\lambda = 780.00$ nm. Suppose we start to slowly increase the wavelength of the light. As λ changes, the photodetector sees "fringes", or bright and dark regions as a function of time:



How much does λ change to go from one dark area (node) to the next ($\Delta\lambda$)? (see picture)

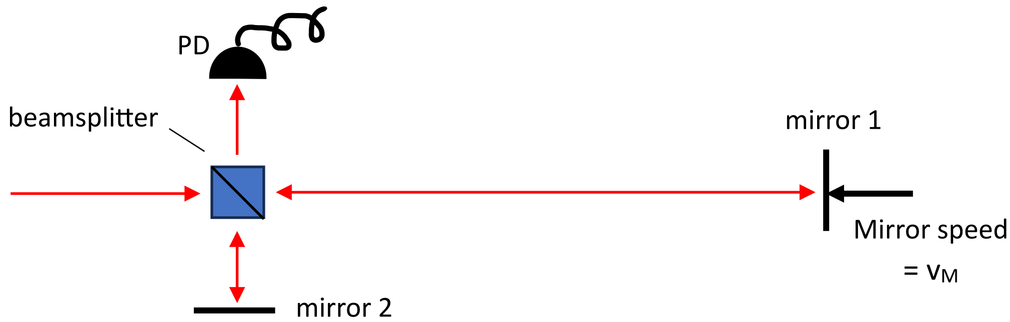
d. Typically, an interferometer is often used as a "ruler" of wavelength (but also sometimes of distance or of velocity). The resolution of an interferometer is usually given as how much the wavelength (or distance) changes between one dark fringe to the next (somewhat similar to a ruler's "resolution" being the distance between the smallest ticks).

Suppose the $L_1 = 1.00$ meter and $L_2 = 10.$ cm. What is the resolution in nanometers?

What is the resolution in Hz? (Be careful! The answer is not $c/\Delta\lambda$.)

e. If you want to get better resolution than in part d, should $\Delta L = L_1 - L_2$ increase or decrease? Explain how this competes with the coherence length of the laser.

4. (4 points) In this problem, we imagine another Michelson interferometer.



In this interferometer, one of the mirrors is moving towards the beamsplitter.

The signal seen by the PD can be understood as the beat note produced by two interfering beams of light at slightly different frequencies: one (beam 2) is the original frequency of the laser, and the other (beam 1) has been shifted to a higher frequency by the Doppler effect.

Recall (from PHYS2310) that the Doppler shift of light reflected off of a moving object (when moving towards the observer) is given by:

$$f_{shifted} = f_0 \left(\frac{v_{light} + v_M}{v_{light} - v_M} \right)$$

a. Using the given variables, find a formula for the beat note frequency that the photodetector measures.

b. For a laser of 550 nm and mirror speed of $v_M = 10$ cm/s, what frequency does the PD measure?

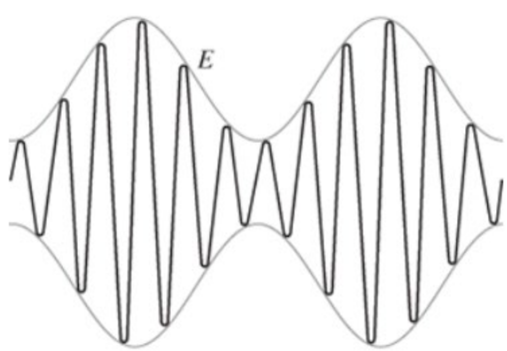
[For examples of this setup used in real life, search for "laser Doppler velocimetry" and read about its applications in measuring aerodynamics, hemodynamics, etc.]

5. (3 points) The figure below shows a carrier of frequency ω_c being amplitude-modulated by a sine wave of frequency ω_m, that is,

$$E = E_0\,(1 + a\cos\omega_m t)\cos\omega_c t$$

Show that this is equivalent to the superposition of three waves of frequencies ω_c, $\omega_c + \omega_m$, and $\omega_c - \omega_m$.

When a number of modulating frequencies are present, we write E as a Fourier series and sum over all values of ω_m. The terms $\omega_c + \omega_m$ constitute what is called the *upper sideband*, and all the $\omega_c + \omega_m$ terms form the *lower sideband*. What bandwidth would you need in order to transmit the complete audible range (20 Hz to 20,000 Hz)?



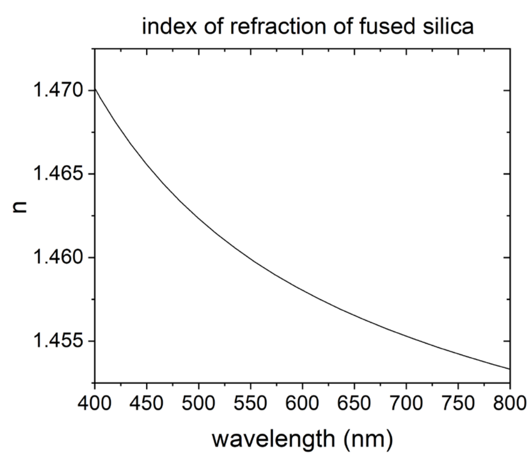
Extra credit (4 points):

a. Show that you can write the group velocity as

$$v_g = \frac{c}{n} + \frac{c}{\lambda}\frac{d(1/n)}{d(1/\lambda)}$$

[*Hint*: first prove that $v_g = d\nu / d(1/\lambda)$.]

b. This plot shows the index of refraction of fused silica (a very pure, non-crystalline form of quartz glass common in optical waveguides) at visible wavelengths.



Using this plot and some basic reasoning, show that the group velocity of visible light in fused silica is slower than its phase velocity.